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Can oil prices help estimate commodity futures prices? The cases of copper and silver

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ABSTRACT

There is an extensive literature on modeling the stochastic process of commodity futures. It has been shown that models with several risk factors are able to adequately fit both the level and the volatility structure of observed transactions with reasonable low errors.

One of the characteristics of commodity futures markets is the relatively short term maturity of their contracts, typically ranging for only a few years. This poses a problem for valuing long term investments that require extrapolating the observed term structure. There has been little work on how to effectively do this extrapolation and in measuring its errors. Cortazar et al. (2008b) propose a multicommodity model that jointly estimates two commodities, one with much longer maturity futures contracts than the other, showing that futures prices of one commodity may be useful information for estimating the stochastic process of another. They implement the procedure using highly correlated commodities like WTI and Brent.

In this paper we analyze using prices of long term oil futures contracts to help estimate long term copper and silver future prices. We start by analyzing the performance of the Cortazar et al. (2008b) multicommodity model, now applied to oil-copper and oil-silver which have much lower correlation than the WTI–Brent contracts. We show that for these commodities with lower correlation the multicommodity model seems not to be effective. We then propose a modified multicommodity model with a much simpler structure which is easier to estimate and that uses the non-stationary long term process of oil to help estimate long term copper and silver futures prices, achieving a much better fit than using available individual or multicommodity models.

Introduction

Natural resource investments are very difficult to value. They usually require large irreversible investments which generate commodity contingent revenues for a long time span that may range for over a decade. To value these investments using the traditional DCF methodology, long term commodity price predictions must be made together with estimates of the risky discount rates.

Commodity prices are extremely volatile. Table 1 presents the volatility of the annualized daily returns of the spot prices of several commodities, showing why forecasting long term prices is such a difficult task. Table 1 also shows how commodity returns are correlated, which, as discussed later, may be helpful to estimate commodity price processes.

Another difficulty for valuing natural resource investments arises because of the many operational and investment flexibilities available to managers who may optimally react to the very different scenarios that these volatile commodity markets induce. Traditional DCF methods tend to ignore flexibilities, undervaluing these investments.

To deal with these problems, there has been an extensive literature which recommends the use of real options theory and no arbitrage arguments for valuing real assets as contingent claims. This approach values an asset as the expected cash flows under a risk-neutral or equivalent martingale measure, and discounts the flows using the risk free rate. By using this approach there is no need to compute risk premiums or expected prices, but only their risk-adjusted estimates. This is particularly convenient for valuing commodity contingent claims when there is a futures market. The price of a futures contract represents the risk adjusted expected commodity price, making price predictions unnecessary.

Currently there is an active futures market for many commodities. Table 2 presents the maturity of futures contracts for a few commodities. It can be seen that for some commodities there are market quotes for futures with long maturities (WTI 9 years) while for others only for very short maturities

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(Copper-Nymex 2 years). The relatively short longest-maturity-contract of some commodities poses a challenge for valuing long term cash flows. For these commodities there is the need for calibrating a futures model to obtain extrapolated prices for maturities beyond existing contracts. These models are also required for valuing operational and investment flexibilities that may be modeled as real options.

As will be discussed in the next section, there are many no arbitrage models for pricing futures prices which differ in the number of stochastic factors, process parameters and risk premium specifications, among others. Some of them do a fairly good job in adjusting observed data, both in level and volatility.

However, there has been little work on how to effectively extrapolate prices and measure their errors. Cortazar et al. (2008b) show that an individual commodity model that closely adjusts existing market quotes may induce increasing errors as model prices are extrapolated well beyond the longest market quote used in the model calibration. To make a better price extrapolation, they propose a multicommodity model that jointly estimates two commodity futures contracts, one with much longer maturity than the other, showing that futures prices of one commodity may be useful information for estimating the stochastic process of another which has shorter maturity contracts. They implement their procedure using highly correlated commodities like WTI and Brent, obtaining a much better extrapolation.

In this paper we analyze using prices of long term oil futures contracts to help estimate long term copper and silver futures prices. We try to take advantage of return correlations among commodities, as shown in Table 1. We start by analyzing the performance of the Cortazar et al. (2008b) multicommodity model now applied to oil-copper and oil-silver, which have much lower correlation than the WTI–Brent contracts. We show that for these commodities with lower correlation, the multicommodity model seems not to be as effective.

We then propose a modified multicommodity model with a much simpler structure which is easier to estimate and that uses the non-stationary long term process of oil to help estimate long term copper and silver futures processes, achieving a much better fit than using available individual or multicommodity models. Our model takes advantage of the Schwartz and Smith (2000) insight that prices move in reaction to two types of shocks. The first one is short term shocks which are stationary, and are responsible for transitory deviations from long term equilibrium. The second type of shocks is responsible for long term movements which have persistent effects and induce a non-stationary price process. We extend the Schwartz and Smith (2000) insight to consider that long term trends in several commodities are related and that there is valuable long term information in one commodity that may be used for estimating the long term process of another. Our results seem to validate our assumption.

The paper is structured as follows. Section 'Futures price models: an introduction' presents an introduction to some futures price models. Section 'The proposed model' presents the proposed model and estimation procedure. Section 'Empirical results for Silver, Copper and Brent' compares the results of using an Individual commodity model, a Multicommodity model (Cortazar et al., 2008b), and our proposed Modified Multicommodity model for extrapolating prices of silver, copper and Brent oil. Finally, section 'Conclusions' concludes.

Table 1
Volatility and correlation of commodity daily returns 01-2004 to 09-2009.

	Volatility (%)	Return correlation		
		Brent (%)	Copper (%)	Silver (%)
WTI	42.5	85.4	37.8	30.3
Brent	38.4		29.7	32.5
Copper	37.3			44.9
Silver	37.8			

Futures price models: an introduction

A commodity futures contract allows two counterparts to commit to a transaction of the underlying commodity at a previously defined date and price. The Spot price is defined as the price of the maturing futures contract. It can be shown by arbitrage arguments that the futures price must equal the expected spot price under a risk-neutral probability distribution. Thus the current futures price discounted at the risk free rate provides a very simple way of valuing a commodity that will be available in the future.

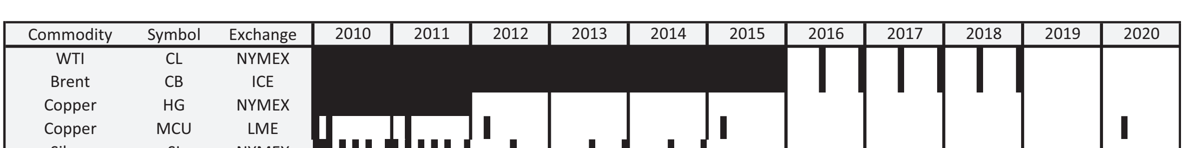
Even though commodity futures markets are becoming increasingly complete with contracts written over more underlying assets and maturities, the need for modeling the behavior of futures prices remains, especially for estimating futures prices of contracts with maturities not traded.

In the last two decades many models have been proposed. Most of them include some risk factors modeled as Brownian motions plus a deterministic trend with different specifications. The simplest of these models considers a geometric Brownian motion with one risk factor and a constant drift (Brennan and Schwartz, 1985).

This simple model has two major drawbacks. First, empirical evidence shows that prices tend to mean revert, which explains why short term futures are more volatile than long maturity ones. The second problem is that the model implies that all futures contracts, regardless of their maturity, are perfectly correlated which defies empirical evidence. To deal with these issues, the model is extended in different ways. First, additional factors are included (Gibson and Schwartz, 1990; Schwartz, 1997; Cortazar and Naranjo, 2006; Cortazar et al., 2008a). These factors may be variables like spot price, convenience yield, interest rates, etc. or, alternatively, may lack any economic interpretation defining a state-space representation (Duffie and Kan, 1996; Dai and Singleton, 2000; Duffie et al., 2000; Cortazar et al., 2007).

A second extension comes from relaxing the constant drift assumption using, for example, an Ornstein–Uhlenbeck mean reverting specification, seasonality or time varying risk

Table 2
Futures contracts for different commodities as of 01-2010 (the copper LME-10.25 year maturity contract was introduced only at the end of 2008 and has relatively low liquidity).



premiums (Laughton and Jacoby, 1993; Sorensen, 2002; Manoliu and Tompaidis, 2002; Cortazar and Schwartz, 2003; Casassus and Collin-Dufresne, 2005).

Finally, different volatility specifications are used. Most of them consider a constant volatility while recently the unspanned stochastic volatility (USV) assumption has been introduced (Trolle and Schwartz, 2007).

We now explain two particular models that are relevant for our purposes. Schwartz and Smith (2000) propose a two-factor model, one representing the long-term equilibrium spot price and the other the short term deviations. Both risk factors explain current spot prices, S_t. The first factor (ξ_t), follows a non-stationary geometric Brownian motion and the second factor (χ_t) a mean-reverting stationary stochastic factor, such that

$$\ln S_t = \xi_t + \chi_t$$

The risk-adjusted dynamics of these factors are:

$$d\xi_t = (\mu_\xi - \lambda_\xi)dt + \sigma_\xi dz_\xi$$

$$d\chi_t = (-\kappa\chi_t - \lambda_\chi)dt + \sigma_\chi dz_\chi$$

with

$$(dz_\xi)(dz_\chi) = \rho_{\xi\chi} dt$$

and λ_i the risk premiums for each risk factor.

Another relevant model is Cortazar et al. (2008b). The paper shows that individual-commodity models, even though they may fit very well existing price observations, sometimes make poor extrapolations for long maturities. To deal with this issue a multicommodity model is presented, where the processes for more than one commodity are jointly modeled. Again one non-stationary state variable is assumed, while two or more stationary state variables are added, some of them shared across different commodities

The paper starts by modeling a single commodity following Cortazar & Naranjo (2006):

$$\ln S_t = 1' x_t + \mu t$$

$$dx_t = (-K x_t - \lambda)dt + \Sigma dw_t$$

Then, the model is extended to include m commodities, all of them sharing p common factors and having k_j factors specific to commodity j such that

$$\begin{bmatrix} Y_1 \\ \vdots \\ Y_i \\ \vdots \\ Y_m \end{bmatrix}_t = \begin{bmatrix} \log S_1 \\ \vdots \\ \log S_i \\ \vdots \\ \log S_m \end{bmatrix} = \begin{bmatrix} \overbrace{ \overbrace{\begin{matrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ \delta_{i1} & \cdots & \delta_{ip} \\ \vdots & & \vdots \\ \delta_{m1} & \cdots & \delta_{mp} \end{matrix}}^{p} \; \overbrace{\begin{matrix} 1 & \cdots & 1 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{matrix}}^{k_1} \; \begin{matrix} \\ \\ \cdots\cdots \\ \\ \\ \end{matrix} \; \overbrace{\begin{matrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 1 & \cdots & 1 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \end{matrix}}^{k_i} \; \begin{matrix} \\ \\ \cdots\cdots \\ \\ \\ \end{matrix} \; \overbrace{\begin{matrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \\ \vdots & & \vdots \\ 1 & \cdots & 1 \end{matrix}}^{k_m} }^{n} \end{bmatrix} X_t + \begin{bmatrix} 1 \\ \vdots \\ \delta_{i1} \\ \vdots \\ \delta_{m1} \end{bmatrix} \mu_t$$

We are now ready to propose our model.

The proposed model

Introduction

The goal of our model is to make better price extrapolations than those from existing individual and multicommodity models, while keeping it tractable. We are concerned with explaining long term, rather than short term, futures prices.

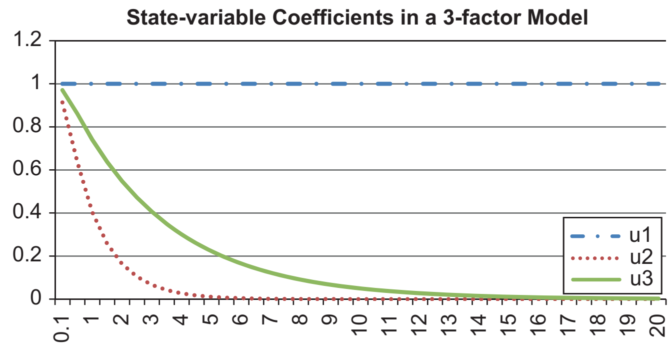


Fig. 1. Risk factor coefficients for different maturities in a silver futures model.

We build on Schwartz and Smith's (2000) insight that long term prices are mainly affected by a non-stationary shock process, while short term prices have transitory deviations from long-term equilibrium which may be modeled using a stationary process. Non-stationarity may arise because these are models of nominal prices subject to inflation. Thus, we should pay particular attention to the non-stationary risk factor if we are concerned with long term prices.

Fig. 1 illustrates this idea. We calibrate a three-factor model for a Silver futures model and set the first factor to have a non-stationary process, while the other two have mean-reverting stationary processes. We recall that the price of a futures contract can be written as

$$F(x_t, t, T) = e^{Ux_t + v}$$

Fig. 1 plots the u_i coefficient from the U vector, corresponding to each of the three factors showing how the relevance of the stationary factors decreases with maturity.

Fig. 2 looks at the same issue in an alternative way. It shows the observed futures prices for Brent on January 6, 2006 and plots our calibrated model using first all risk factors, and then setting the two stationary factors to zero. It can be seen that the model using only the stationary factor performs increasingly well, the longer the maturity.

Thus, for effectively extrapolating long term prices we are mainly concerned with calibrating the process for the non-stationary risk factor. Our problem is how to do this without any market information for a commodity. To deal with this problem we extend Schwartz and Smith (2000) and conjecture that a significant portion of the trend that explains long term futures prices are due to some common macroeconomic factors that affect all commodities. If this is the case, the relevant long term trend for one commodity may be elicited from the long term process of another commodity which does trade long-term futures. Our results in section 'Empirical results for silver, copper and Brent' are consistent with this conjecture.

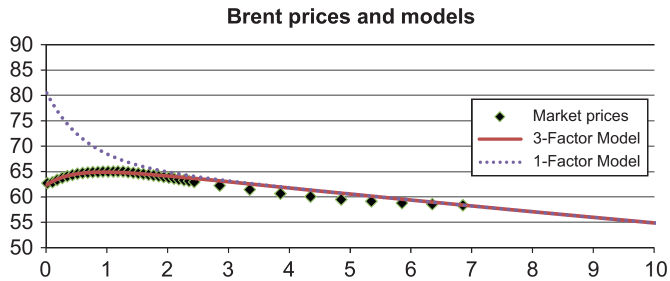


Fig. 2. Brent futures on January 6, 2006: observed prices, 3-factor model, and 1-factor model (using only the non-stationary factor).

The modified multicommodity (MM) model

We now present a general m commodity model. This model can be seen as a modification of the Cortazar et al. (2008b) Multicommodity model and will be called the Modified Multicommodity (MM) model. Later, 2-commodity versions of this model will be estimated to extrapolate copper, silver and Brent futures prices and compare their performance to existing alternatives.

We start by defining Y_i as the logarithm of the spot price of commodity i, S_i:

$$Y_t=\begin{bmatrix}Y_1\\ \vdots\\ Y_i\\ \vdots\\ Y_m\end{bmatrix}_t=\log\begin{bmatrix}S_1\\ \vdots\\ S_i\\ \vdots\\ S_m\end{bmatrix}_t,\quad i=1,\dots,m \text{ commodities}$$

Let X_t be a vector of n state variables, which explains all m commodity prices, with the first m state variables the non-stationary risk factors for the m-commodities. We assume that these non-stationary risk factors are different for each commodity, but share the same process parameters. Also for each of the m-commodities we assume there are nf_i commodity-specific stationary factors, each with its own independent process. Then $n=\sum_i(nf_i+1)$

and

$Y_t=HX_t$

$$H=\begin{bmatrix}\overbrace{1\ \dots\ 0\ \cdots\ 0}^{m} & \overbrace{1\ \dots\ 1}^{nf_1} & \dots & \overbrace{0\ \dots\ 0}^{nf_i} & \dots & \overbrace{0\ \dots\ 0}^{nf_m}\\ \vdots\quad\ \ \vdots\quad\ \ \vdots & \vdots\quad\ \vdots & & \vdots\quad\ \vdots & & \vdots\quad\ \vdots\\ 0\ \dots\ 1\ \cdots\ 0 & 0\ \dots\ 0 & \dots & 1\ \dots\ 1 & \dots & 0\ \dots\ 0\\ \vdots\quad\ \ \vdots\quad\ \ \vdots & \vdots\quad\ \vdots & & \vdots\quad\ \vdots & & \vdots\quad\ \vdots\\ 0\ \dots\ 0\ \cdots\ 1 & 0\ \dots\ 0 & \dots & 0\ \dots\ 0 & \dots & 1\ \dots\ 1\end{bmatrix}$$

We then assume a generalized-Vasicek (Vasicek, 1977) risk-adjusted process with a vector of constant risk premiums λ for the state variables:

$$dX_t=(\mu-\lambda-KX_t)\,dt+\sigma\,dw_t$$

where dw_t is a $n\times 1$ vector of correlated Brownian motion increments such that

$$(dw_t)'(dw_t)=\Theta\,dt$$

$$\mu=\begin{bmatrix}\mu_1\\ \vdots\\ \mu_1\\ 0\\ \vdots\\ 0\end{bmatrix}\begin{matrix}\left.\vphantom{\begin{matrix}1\\1\\1\end{matrix}}\right\}m\\ \vphantom{\begin{matrix}0\\0\\0\end{matrix}}\end{matrix},\quad \lambda=\begin{bmatrix}\lambda_1\\ \vdots\\ \lambda_1\\ \lambda_{m+1}\\ \vdots\\ \lambda_n\end{bmatrix},$$

$$K=\begin{bmatrix}0 & \dots & 0 & 0 & \dots & 0\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ 0 & \dots & 0 & 0 & \dots & 0\\ 0 & \dots & 0 & k_{m+1} & \dots & 0\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ 0 & \dots & 0 & 0 & \dots & k_n\end{bmatrix},$$

$$\Sigma=\begin{bmatrix}\sigma_1 & \dots & 0 & 0 & \dots & 0\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ 0 & \dots & \sigma_1 & 0 & \dots & 0\\ 0 & \dots & 0 & \sigma_{m+1} & \dots & 0\\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots\\ 0 & \dots & 0 & 0 & \dots & \sigma_n\end{bmatrix}$$

$$\Theta=\begin{bmatrix}1 & \rho_{12} & \dots & \rho_{1n}\\ \rho_{21} & 1 & \dots & \rho_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \rho_{n1} & \rho_{n2} & \dots & 1\end{bmatrix}$$

Then, it can be easily shown that the price of a futures contract, at time t, for delivery of the underlying at time T, is

$$F(S_t,t,T)=\exp\left(x_1(t)+\sum_{j=2}^{n}e^{-k_j(T-t)}x_j(t)+\left(\mu-\lambda_1+\frac{1}{2}\sigma_1^2\right)(T-t)\right.$$
$$\left.-\sum_{j=2}^{n}\frac{1-e^{-k_j(T-t)}}{k_j}\lambda_j+\frac{1}{2}\sum_{j,l\neq 1}^{n}\sigma_j\sigma_l\rho_{jl}\frac{1-e^{-(k_j+k_l)(T-t)}}{k_j+k_l}\right)$$

Notice that the logarithm of the future is linear in the state variables, which is very convenient for estimating the model. Also that sharing the process parameters does not assume cointegration between the commodity price series. Cointegration requires that a linear combination of two series reduces the order of integration. Instead of assuming cointegration, we make a much weaker assumption that the process parameters (mean drifts and volatilities) are related between commodities, but we do not impose any restriction on the stochastic shocks. In other words we assume that, provided we do not have long term prices for a given commodity, it might be better to use the information on the drift and volatility of other commodities (that may be partially related to some common macroeconomic factors), than to extrapolate this drift from short term prices. We do not assert that a linear combination of these two commodities is stationary and recognize that probably it is not because there might be other factors (that could be non-stationary) that could explain the differing behavior of the prices of two commodities.

Some extensions to the above model could be easily made. For example, if we had independent information on the relationship of the long term trends between commodities, μ_1 could be replaced by $\alpha_i\mu_1$. By the same token, σ_1 could be replaced by $\beta_i\sigma_1$. Without this information we are assuming $\alpha_j=\beta_j=1$. Also we could include a variable vector of risk premiums:

$$\lambda_x=\lambda+\Lambda x$$

where Λ is a matrix that relates the market price of risk to the state variables, like in Casassus and Collin-Dufresne (2005), and the model can still be solved.

The estimation of a simple 2-commodity MM model

In this section we show how to formulate and estimate a simple version of the general model described before, but now applied to two commodities: one with long maturity contracts and the other with shorter ones. We are aiming to use the information of the long-maturity commodity to help estimate long term futures prices for the other commodity.

Let us assume the model has $n=4$ state variables, two for each commodity. Each commodity is assumed to have a non-stationary variable and a stationary one. In this setting the logarithm of the spot price vector, Y_t, is related to the vector X_t with the 4 state variables, as follows:

$$\begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} X_{t(4\times 1)}$$

$$dx_t = (\mu - \lambda - Kx_t)\,dt + \Sigma\, dw_t$$

where dw_t is a $n \times 1$ vector of correlated Brownian motion increments such that

$$(dw_t)'(dw_t) = \Theta\, dt$$

$$\mu = \begin{bmatrix} \mu \\ \mu \\ 0 \\ 0 \end{bmatrix}, \quad \lambda = \begin{bmatrix} \lambda_1 \\ \lambda_1 \\ \lambda_3 \\ \lambda_4 \end{bmatrix}$$

$$K = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}$$

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_1 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 \\ 0 & 0 & 0 & \sigma_4 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \rho_{14} \\ \rho_{21} & 1 & \rho_{23} & \rho_{24} \\ \rho_{31} & \rho_{32} & 1 & \rho_{34} \\ \rho_{41} & \rho_{42} & \rho_{43} & 1 \end{bmatrix}$$

To estimate a 2-commodity model Cortazar et al. (2008b) propose to jointly calibrate all parameters using an incomplete panel Kalman Filter, showing that, at least for highly correlated commodities, very good results may be obtained, at the expense of rather heavy computing requirements. In their model both commodities share some factors while having others that are commodity-specific.

Our proposed model, on the other hand, only shares the parameters of the non-stationary factor process, and we believe this can be better estimated using only the commodity that exhibits long maturity contracts. Thus we will dramatically simplify the model calibration by using a two-stage estimation process. We first individually estimate the long-maturity-contract commodity process. Then, in a second step, we individually estimate the short-maturity-contract commodity restricting the parameters of its non-stationary factor to be equal to the values obtained in the first step for the other commodity.

For each of the steps we use the Kalman Filter and maximum likelihood to obtain state variables and optimal parameter values.

Empirical results for silver, copper and Brent

The data

We use the WTI oil futures contracts to help estimate futures prices of three commodities: Silver, Copper and Brent. For each commodity we use daily futures contracts traded at Nymex, LME, or IPE markets. The contracts are classified into 4 panels for each commodity, one in-sample and the other three out-of-sample sets. Fig. 3 illustrates these 4 panels.

The In-Sample panel will be used to estimate model parameters and state variables.

The Out-of-Sample-1 panel will be used only to estimate state variables, but not model parameters.

Finally, the Out-of-Sample-2 and Out-of-Sample-3 panels will not be used at all to estimate state variables or model parameters, and will be helpful for measuring the extrapolation performance of the alternative models.

For Silver we use daily futures contracts from 1 month to 5 years traded at NYMEX. The In-Sample panel considers contracts with maturities up to 2 years traded from 01-2004 to 12-2007. The Out-of-Sample-1 set considers contracts with maturities up to 2 years traded from 01-2008 to 10-2009. The Out-of-Sample-2 data considers contracts with maturities of more than 2 years traded from 01-2004 to 12-2007. The Out-of-Sample-3 set considers contracts with maturities of more than 2 years traded from 01-2008 to 10-2009.

For Copper we use daily futures contracts from 1 month to 10.25 years traded at NYMEX and LME. The In-Sample panel considers contracts with maturities up to 2 years traded from 01-2004 to 12-2007. The Out-of-Sample-1 data considers contracts with maturities up to 2 years traded from 01-2008 to 10-2009. The Out-of-Sample-2 set considers contracts with maturities of more than 2 years traded from 01-2004 to 12-2007. The Out-of-Sample-3 data considers contracts with maturities of more than 2 years traded from 01-2008 to 10-2009.

For Brent we use daily futures contracts from 1 month to 7 years traded at IPE. The selection of contracts and dates is the

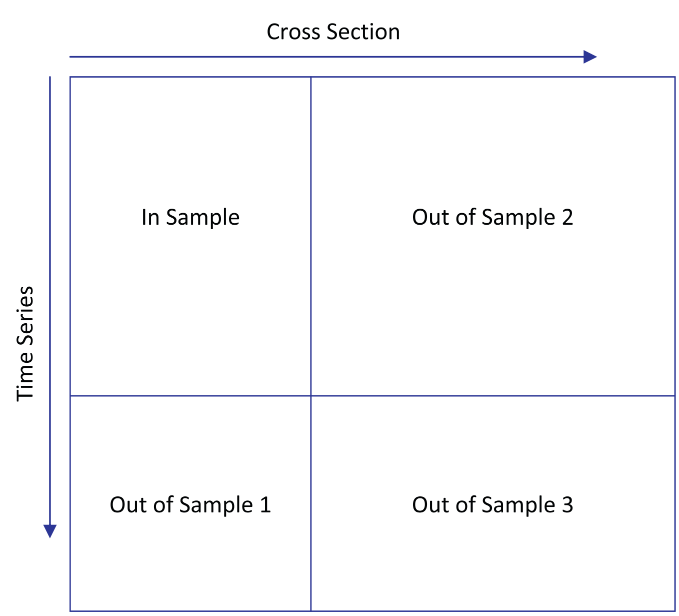


Fig. 3. The four data panel constructed for each commodity.

same as in Cortazar et al. (2008b). The In-Sample panel considers contracts with maturities up to 2.5 years traded from 01-2001 to 12-2004. The Out-of-Sample-1 data considers contracts with maturities up to 2.5 years traded from 01-2005 to 12-2006. The Out-of-Sample-3 data considers contracts with maturities of more than 2.5 years traded from 01-2005 to 12-2006. Cortazar et al. (2008b) call this last panel "Extreme Out-of-Sample" but do not report results for an Out-of-Sample-2 panel.

All our models use as the long-maturity commodity WTI futures traded at Nymex.

Model parameters

For each of the three commodities (Silver, Copper and Brent) we will calibrate three models. First, an individual model that uses only one commodity. Second, the Cortazar et al. (2008b) Multicommodity model that jointly calibrates the given commodity with WTI. Finally our proposed Modified Multicommodity model (MM), which uses the WTI only to obtain the parameters of the non-stationary factor process and then estimates individually each commodity restricting the non-stationary factor process parameters.

We estimate the Individual and the MM models for Silver and Copper with three factors, and for Brent with four. The Multicommodity model for each commodity also has three or four factors, respectively. Tables 3–5 show the parameter values and standard deviations for each of the models including Silver, Copper and Brent, respectively.

It can be seen that the parameters are statistically significant and are reasonable in magnitude and sign.

Results

In what follows we present the results for silver, copper and Brent, using the alternative models. To compare the relative performance of the models we use a procedure adapted from Schwartz (1997). We must keep in mind, however, that the purpose of our paper is not to find a forecasting model to predict

Table 3
Parameter values and standard deviations for Individual, Multicommodity and Modified Multicommodity models for Silver.

Parameter	Individual	Multicommodity	MM—modified multicommodity
k_2	0.904 (0.004)	0.275 (0.001)	0.770 (0.000)
k_3	0.300 (0.003)	0.798 (0.003)	0.316 (0.000)
k_4	–	0.131 (0.001)	–
k_5	–	0.412 (0.002)	–
σ_1	0.219 (0.000)	0.359 (0.000)	0.275 (0.008)
σ_2	0.165 (0.001)	0.151 (0.0101)	0.145 (0.001)
σ_3	0.197 (0.000)	0.216 (0.001)	0.220 (0.000)
σ_4	–	0.453 (0.002)	–
σ_5	–	0.304 (0.001)	–
ρ_{12}	−0.133 (0.002)	0.121 (0.001)	−0.177 (0.008)
ρ_{13}	−0.217 (0.001)	−0.122 (0.001)	−0.217 (0.000)
ρ_{14}	–	0.046 (0.001)	–
ρ_{15}	–	0.139 (0.001)	–
ρ_{23}	−0.341 (0.003)	0.149 (0.005)	−0.528 (0.001)
ρ_{24}	–	−0.154 (0.007)	–
ρ_{25}	–	−0.107 (0.008)	–
ρ_{34}	–	−0.177 (0.001)	–
ρ_{35}	–	0.111 (0.008)	–
ρ_{45}	–	−0.138 (0.001)	–
λ_1	0.102 (0.011)	0.054 (0.000)	0.197 (0.001)
λ_2	−0.413 (0.069)	0.318 (0.001)	−0.405 (0.054)
λ_3	0.219 (0.061)	0.276 (0.001)	0.162 (0.068)
λ_4	–	0.105 (0.000)	–
λ_5	–	0.185 (0.001)	–
μ	0.062 (0.010)	0.005 (0.000)	0.164 (0.002)
ξ_{CL}	–	0.018 (0.000)	–
ξ_{SI}	0.002 (0.000)	0.011 (0.000)	0.002 (0.000)
δ	–	0.997 (0.004)	–

Table 4
Parameter values and standard deviations for Individual, Multicommodity and Modified Multicommodity models for copper.

Parameter	Individual	Multicommodity	MM—modified multicommodity
k_2	2.853 (0.225)	0.797 (0.033)	1.659 (0.087)
k_3	0.585 (0.031)	0.483 (0.014)	0.386 (0.007)
k_4	–	0.306 (0.001)	–
k_5	–	0.100 (0.000)	–
σ_1	0.363 (0.013)	0.260 (0.000)	0.275 (0.008)
σ_2	0.199 (0.020)	0.424 (0.007)	0.315 (0.015)
σ_3	0.410 (0.016)	0.362 (0.007)	0.727 (0.042)
σ_4	–	0.500 (0.005)	–
σ_5	–	0.287 (0.003)	–
ρ_{12}	−0.227 (0.112)	0.460 (0.062)	0.395 (0.114)
ρ_{13}	−0.250 (0.020)	−0.186 (0.003)	−0.484 (0.035)
ρ_{14}	–	−0.500 (0.008)	–
ρ_{15}	–	−0.172 (0.003)	–
ρ_{23}	0.099 (0.033)	−0.201 (0.016)	−0.645 (0.009)
ρ_{24}	–	0.209 (0.003)	–
ρ_{25}	–	−0.404 (0.010)	–
ρ_{34}	–	−0.085 (0.001)	–
ρ_{35}	–	−0.077 (0.002)	–
ρ_{45}	–	−0.448 (0.006)	–
λ_1	0.238 (0.030)	0.149 (0.000)	0.197 (0.001)
λ_2	0.320 (0.115)	0.363 (0.007)	0.531 (0.147)
λ_3	0.388 (0.160)	0.271 (0.003)	−0.754 (0.219)
λ_4	–	0.138 (0.002)	–
λ_5	–	0.051 (0.000)	–
μ	0.057 (0.028)	0.119 (0.000)	0.164 (0.002)
ξ_{CL}	–	0.013 (0.000)	–
ξ_{HG}	0.024 (0.000)	0.011 (0.000)	0.024 (0.000)
δ	–	0.994 (0.035)	–

Table 5
Parameter values and standard deviations for Individual, Multicommodity and Modified Multicommodity models for Brent.

Parameter	Individual	Multicommodity	MM—modified multicommodity
k_2	0.460 (0.006)	0.384 (0.005)	0.451 (0.000)
k_3	0.697 (0.005)	0.911 (0.002)	0.927 (0.000)
k_4	6.951 (0.094)	0.435 (0.001)	6.951 (0.026)
σ_1	0.252 (0.003)	0.196 (0.000)	0.424 (0.000)
σ_2	0.885 (0.007)	0.178 (0.003)	0.886 (0.000)
σ_3	1.000 (0.006)	0.336 (0.003)	1.000 (0.000)
σ_4	0.121 (0.005)	0.080 (0.003)	0.178 (0.001)
ρ_{12}	−0.567 (0.003)	−0.389 (0.022)	−0.567 (0.001)
ρ_{13}	0.525 (0.003)	0.370 (0.020)	0.525 (0.000)
ρ_{14}	−0.055 (0.002)	−0.192 (0.017)	−0.110 (0.000)
ρ_{23}	−0.965 (0.011)	−0.589 (0.004)	−0.965 (0.000)
ρ_{24}	0.275 (0.025)	0.180 (0.007)	0.275 (0.000)
ρ_{34}	−0.310 (0.023)	−0.087 (0.006)	−0.310 (0.000)
λ_1	0.051 (0.001)	0.022 (0.000)	0.079 (0.000)
λ_2	−0.068 (0.359)	0.046 (0.058)	0.583 (0.279)
λ_3	0.213 (0.450)	−0.003 (0.138)	0.692 (0.356)
λ_4	− 0.179 (0.080)	0.017 (0.000)	−0.179 (0.094)
μ	0.000 (0.001)	0.002 (0.000)	0.001 (0.000)
ξ_{CL}	–	0.005 (0.000)	–
ξ_{CB}	0.003 (0.000)	0.008 (0.000)	0.003 (0.000)
δ_{11}	–	1.000 (0.001)	–
δ_{12}	–	1.000 (0.000)	–
δ_{13}	–	1.000 (0.002)	–
δ_{14}	–	–	–
δ_{21}	–	1.000 (0.001)	–
δ_{22}	–	1.000 (0.000)	–
δ_{23}	–	1.000 (0.000)	–
δ_{24}	–	1.000 (0.003)	–

prices, but we are rather interested in the cross-section extrapolation of prices when there is information on short term prices and we lack information on current long-term contract prices. Thus we will emphasize cross-section over time-series tests.

First, for each of the commodities we compare the cross-section behavior of the models for two dates. The first date is selected from the set used to calibrate the model parameters, thus short term contracts for that day are from the in-sample set, and long term contracts are from the out-of-sample 2 set (see Fig. 3). The second date is selected from the set not used to calibrate model parameters, thus contracts belong to the out-of-sample 1 or to the out-of-sample 3, depending on their maturity. In this way we can illustrate the extrapolation behavior of each model for both types of dates.

If a full statistical comparison among the models is to be performed, adapting a procedure from the literature could be done (Pesaran and Deaton, 1978; Davidson and Mackinnon, 1981; Clark and West, 2007; Diebold and Mariano, 1995). However, as Schwartz (1997) points out, a statistical comparison between these types of models is not straightforward, in particular because tests are not easily adapted to this framework and some of the models are nested while others are not. Moreover, as was stated previously, in this paper we are basically interested in the cross-section behavior of the models.

We then calculate the mean errors and the root mean square errors for the 4 data panels and the 3 alternative models. Notice that our out-of-sample 2 and out-of-sample 3 cross-section analyses are really extrapolation exercises in which all contracts have long maturities, while the Schwartz (1997) cross-section analysis is an interpolation because out-of-sample contracts have shorter maturities than some of the in-sample data. Also, our reported out-of-sample errors really represent an upper bound, because continuous updating of the parameters should provide lower errors. Given that daily recalculation of all model parameters is very high in computational resources, we follow Schwartz (1997) and provide upper and lower bounds for cross-section errors. Thus, we report for each long maturity contract in the out-of-sample 3 data set, the lower and upper bounds for the errors in each model. The upper bound is obtained using the in-sample model parameters, while the lower bound is computed with model parameters updated including the out-of-sample 1 set. As Schwartz (1997) points out a daily updating of model parameters should provide errors between these bounds.

We now show the results of comparing the three models for each commodity.

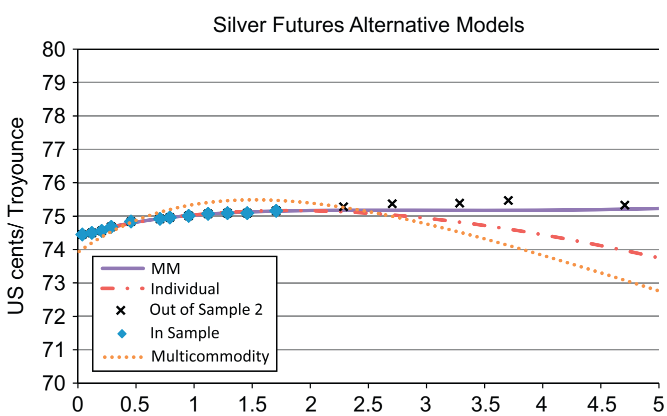


Fig. 4. Silver futures prices and models for 04-13-2004.

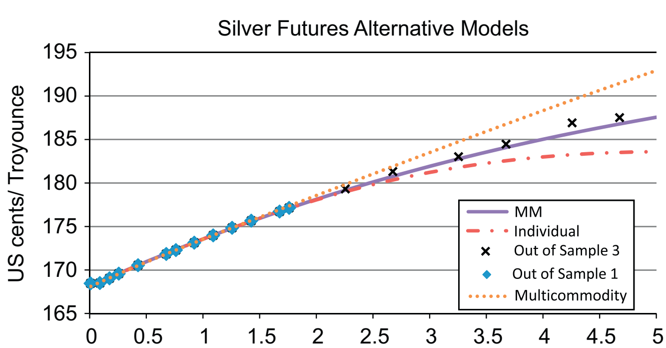


Fig. 5. Silver futures prices and models for 04-25-2008.

Results for Silver

Figs. 4 and 5 compare the adjustment of the three models for two dates which extrapolate prices from the out-of-sample 2 and the out-of-sample 3 panels. The figures show that for both dates, extrapolated prices from the MM model are much better than those from the alternative models.

Table 6 presents a summary of the errors (ME and RMSE) for the 4 data panels and all alternative models. The first thing that can be noted is that the Cortazar et al. (2008b) multicommodity model does not seem to be effective for Silver, a commodity with a low correlation with WTI. Second, our Modified Multicommodity (MM) model performs very well making much better price extrapolations than those of the other two alternative models.

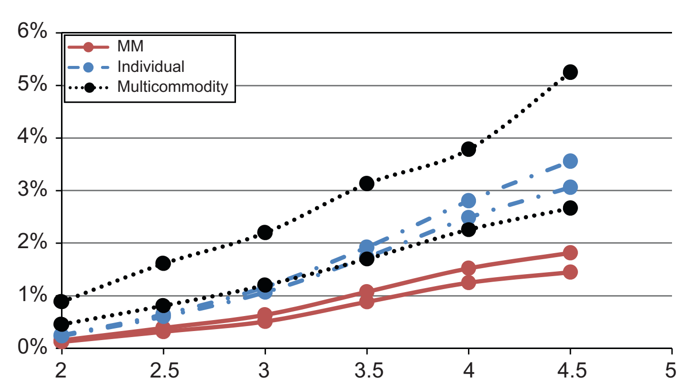


Fig. 6. Silver futures cross-section out-of-sample upper and lower bounds for errors for contracts with different maturities.

Table 6
Mean errors (ME) and root mean square errors (RMSE) for silver under alternative models and data panels.

	ME individual (%)	ME multicommodity (%)	ME MM (%)	RMSE individual (%)	RMSE multicommodity (%)	RMSE MM (%)
In sample	0.00	0.00	0.00	0.09	0.22	0.09
Out of sample 1	0.00	0.01	0.00	0.05	0.18	0.04
Out of sample 2	−1.05	0.52	−0.35	1.68	2.41	1.15
Out of sample 3	−1.65	1.48	−0.82	2.08	2.22	1.10

Finally, Fig. 6 shows the upper and lower bounds for the cross-section errors of long maturity contracts for each of the models. It can be seen that the MM model clearly outperforms the other models for all contracts while there are no conclusive results between the Individual and the Multicommodity models.

Results for Copper

Figs. 7 and 8 show the model adjustments for the Out-of-Sample 2 and the Out-of-Sample 3 Copper panels. Again, the MM model performs much better than the alternative models.

Table 7 presents the ME and RMSE errors for the 4 panels and the alternative models. It can be noted that again our MM-Modified Multicommodity model performs very well making much better price extrapolations than the other alternative models. Also that in this case extrapolating prices using the Multicommodity model is slightly better than using the individual model. Notice that the WTI-copper correlation is higher than the WTI-silver correlation (even though much lower than the WTI–Brent).

Fig. 9 shows the upper and lower bounds for the cross-section errors of long maturity Copper Futures contracts for each of the models. It can be seen that the MM model clearly outperforms the other models for all contracts while the multicommodity model seems slightly better than the individual models.

Results for Brent

Table 8 shows the results of all three models for extrapolating Brent prices. As discussed previously, for highly correlated commodities, like WTI–Brent, the Cortazar et al. (2008b) Muticommodity model is very good at price extrapolation.

Conclusions

There are many stochastic models of futures prices that do a very good job at fitting price levels and volatility structures. Good models require several risk factors with some of them non-stationary and some mean reverting, to fit the stochastic behavior of commodity futures.

One remaining problem in the literature is how to extrapolate futures prices beyond the longest maturity that trades in the market. This problem was partially dealt with in Cortazar et al. (2008b) who propose a multicommodity model that uses the information of one commodity that has long maturity contracts to help estimate another commodity with shorter contracts. They show their model behaves well for a pair of highly correlated commodities like WTI–Brent.

This paper deals with the same problem but now for lower correlated commodities like WTI-Silver and WTI-Copper showing that the multicommodity model is not effective in these cases. It then proposes a modified multicommodity model that only shares the parameters of the non-stationary risk factor process, showing that WTI prices help to estimate long-term Silver and Copper prices in a much more effective way. Future research could explore models including other commodities with low correlation and comparing the effectiveness of the proposed model against other benchmarks like univariate Garch model or some simple vector–error–correction models.

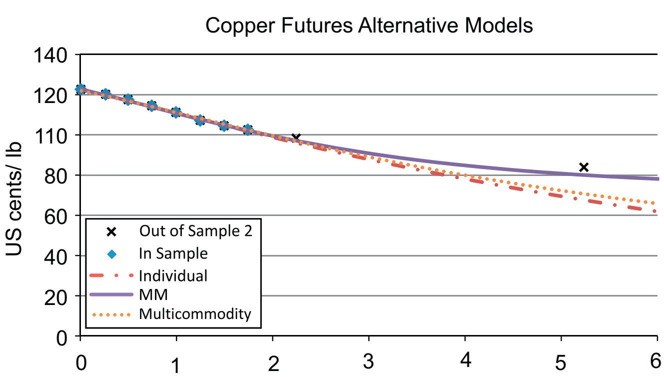


Fig. 7. Copper futures prices and models for 06-30-2004.

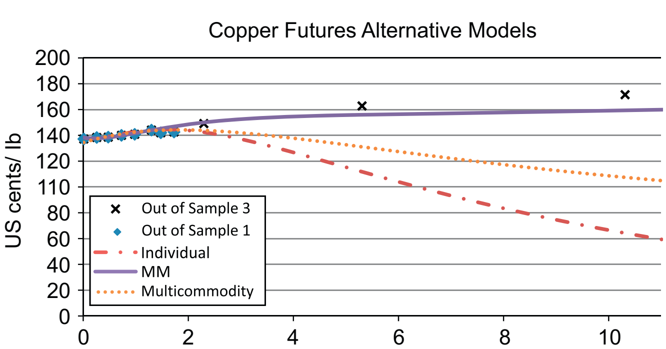


Fig. 8. Copper futures prices and models for 12-05-2008.

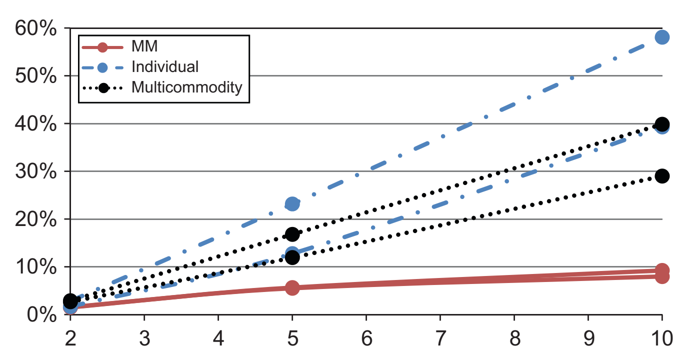


Fig. 9. Copper futures cross-section out-of-sample upper and lower bound errors for different maturities.

Table 7
Mean errors (ME) and root mean square errors (RMSE) for copper under alternative models and data panels.

	ME individual (%)	ME multicommodity (%)	ME MM (%)	RMSE individual (%)	RMSE multicommodity (%)	RMSE MM (%)
In sample	0.03	−0.11	0.00	2.25	2.33	2.26
Out of sample 1	0.02	−0.34	−0.01	0.62	0.84	0.60
Out of sample 2	−5.08	−3.71	3.61	10.86	9.61	8.70
Out of sample 3	−22.75	−16.27	−2.31	31.14	21.67	5.71

Table 8
Mean errors (ME) and root mean square errors (RMSE) for Brent under alternative models and data panels. Multicommodity values are those reported in Cortazar et al. (2008b).

	ME individual (%)	ME multicommodity (%)	ME MM (%)	RMSE individual (%)	RMSE multicommodity (%)	RMSE MM (%)
In sample	0.00	−0.01	0.00	0.31	0.80	0.31
Out of sample 1	1.17	0.36	−0.10	2.86[a]	1.06	0.32
Out of sample 3[b]	4.26	1.56	−1.60	5.15	1.74	2.39

[a] We report the number published in Cortazar et al. (2008b), even though redoing the calculation we obtain 0.11% which seems more reasonable.
[b] Cortazar et al. (2008b) report this result under the name *''Extreme out of sample''*.

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